

**Friday 13 January 2012 – Morning**

**A2 GCE MATHEMATICS**

**4726** Further Pure Mathematics 2

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4726
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 Given that  $f(x) = \ln(\cos 3x)$ , find  $f'(0)$  and  $f''(0)$ . Hence show that the first term in the Maclaurin series for  $f(x)$  is  $ax^2$ , where the value of  $a$  is to be found. [4]

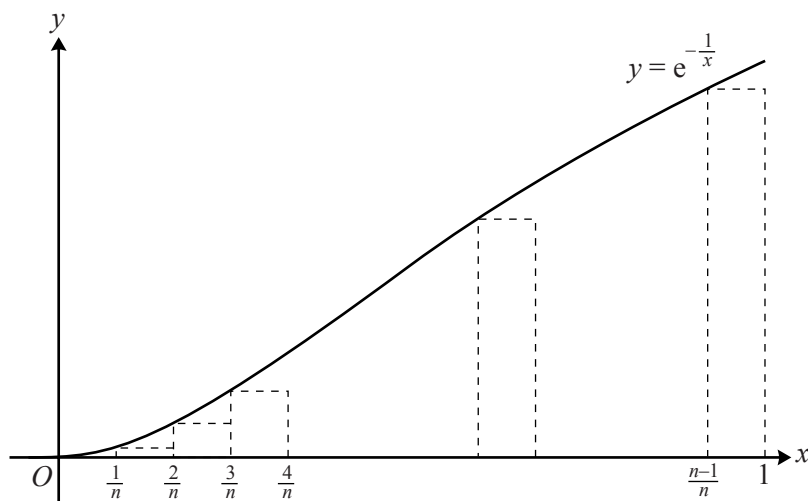
- 2 By first completing the square in the denominator, find the exact value of

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2 - 4x + 5} dx.$$

[5]

- 3 Express  $\frac{2x^3 + x + 12}{(2x - 1)(x^2 + 4)}$  in partial fractions. [7]

4



The diagram shows the curve  $y = e^{-\frac{1}{x}}$  for  $0 < x \leq 1$ . A set of  $(n - 1)$  rectangles is drawn under the curve as shown.

- (i) Explain why a lower bound for  $\int_0^1 e^{-\frac{1}{x}} dx$  can be expressed as

$$\frac{1}{n} \left( e^{-n} + e^{-\frac{n}{2}} + e^{-\frac{n}{3}} + \dots + e^{-\frac{n}{n-1}} \right).$$

[2]

- (ii) Using a set of  $n$  rectangles, write down a similar expression for an upper bound for  $\int_0^1 e^{-\frac{1}{x}} dx$ . [2]
- (iii) Evaluate these bounds in the case  $n = 4$ , giving your answers correct to 3 significant figures. [2]
- (iv) When  $n \geq N$ , the difference between the upper and lower bounds is less than 0.001. By expressing this difference in terms of  $n$ , find the least possible value of  $N$ . [3]

- 5 It is given that  $f(x) = x^3 - k$ , where  $k > 0$ , and that  $\alpha$  is the real root of the equation  $f(x) = 0$ . Successive approximations to  $\alpha$ , using the Newton-Raphson method, are denoted by  $x_1, x_2, \dots, x_n, \dots$ .

(i) Show that  $x_{n+1} = \frac{2x_n^3 + k}{3x_n^2}$ . [2]

- (ii) Sketch the graph of  $y = f(x)$ , giving the coordinates of the intercepts with the axes. Show on your sketch how it is possible for  $|\alpha - x_2|$  to be greater than  $|\alpha - x_1|$ . [3]

It is now given that  $k = 100$  and  $x_1 = 5$ .

- (iii) Write down the exact value of  $\alpha$  and find  $x_2$  and  $x_3$  correct to 5 decimal places. [3]

- (iv) The error  $e_n$  is defined by  $e_n = \alpha - x_n$ . By finding  $e_1, e_2$  and  $e_3$ , verify that  $e_3 \approx \frac{e_2^3}{e_1^2}$ . [3]

- 6 (i) Prove that the derivative of  $\cos^{-1}x$  is  $-\frac{1}{\sqrt{1-x^2}}$ . [3]

A curve has equation  $y = \cos^{-1}(1 - x^2)$ , for  $0 < x < \sqrt{2}$ .

- (ii) Find and simplify  $\frac{dy}{dx}$ , and hence show that

$$(2 - x^2) \frac{d^2y}{dx^2} = x \frac{dy}{dx}. \quad [5]$$

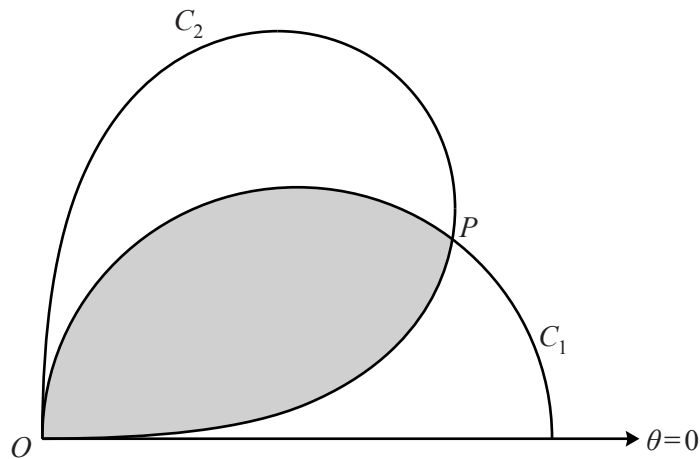
- 7 (i) Given that  $y = \sinh^{-1}x$ , prove that  $y = \ln(x + \sqrt{x^2 + 1})$ . [3]

- (ii) It is given that  $x$  satisfies the equation  $\sinh^{-1}x - \cosh^{-1}x = \ln 2$ . Use the logarithmic forms for  $\sinh^{-1}x$  and  $\cosh^{-1}x$  to show that

$$\sqrt{x^2 + 1} - 2\sqrt{x^2 - 1} = x.$$

Hence, by squaring this equation, find the exact value of  $x$ . [5]

[Questions 8 and 9 are printed overleaf.]



The diagram shows two curves,  $C_1$  and  $C_2$ , which intersect at the pole  $O$  and at the point  $P$ . The polar equation of  $C_1$  is  $r = \sqrt{2} \cos \theta$  and the polar equation of  $C_2$  is  $r = \sqrt{2} \sin 2\theta$ . For both curves,  $0 \leq \theta \leq \frac{1}{2}\pi$ . The value of  $\theta$  at  $P$  is  $\alpha$ .

(i) Show that  $\tan \alpha = \frac{1}{2}$ . [2]

(ii) Show that the area of the region common to  $C_1$  and  $C_2$ , shaded in the diagram, is  $\frac{1}{4}\pi - \frac{1}{2}\alpha$ . [7]

9 (i) Show that  $\tanh(\ln n) = \frac{n^2 - 1}{n^2 + 1}$ . [2]

It is given that, for non-negative integers  $n$ ,  $I_n = \int_0^{\ln 2} \tanh^n u \, du$ .

(ii) Show that  $I_n - I_{n-2} = -\frac{1}{n-1} \left(\frac{3}{5}\right)^{n-1}$ , for  $n \geq 2$ . [3]

(iii) Find the value of  $I_3$ , giving your answer in the form  $a + \ln b$ , where  $a$  and  $b$  are constants. [4]

(iv) Use the method of differences on the result of part (ii) to find the sum of the infinite series

$$\frac{1}{2} \left(\frac{3}{5}\right)^2 + \frac{1}{4} \left(\frac{3}{5}\right)^4 + \frac{1}{6} \left(\frac{3}{5}\right)^6 + \dots$$

[2]

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